Doppler effect and Hubble effect in different models of space-time in the case of auto-parallel motion of the observer

Sawa Manoff
Bulgarian Academy of Sciences
Institute for Nuclear Research and Nuclear Energy
Department of Theoretical Physics
Laboratory of Solitons, Coherency, and Geometry
1784 Sofia - Bulgaria

E-mail address: smanov@inrne.bas.bg

Abstract

Doppler effect and Hubble effect in different models of space-time in the case of auto-parallel motion of the observer are considered. The Doppler effect and shift frequency parameter are specialized for the case of auto-parallel motion of the observer. The Hubble effect and shift frequency parameter are considered for the same case. It is shown that by the use of the variation of the shift frequency parameter during a time period, considered locally in the proper frame of reference of an observer, one can directly determine the centrifugal (centripetal) relative velocity and acceleration as well as the Coriolis relative velocity and acceleration of an astronomical object moving relatively to the observer. All results are obtained on purely kinematic basis without taking into account the dynamic reasons for the considered effect.

PACS numbers: 98.80.Jk; 98.62.Py; 04.90.+e; 04.80.Cc;

Contents

1 Introduction

1. Modern problems of relativistic astrophysics as well as of relativistic physics (dark matter, dark energy, evolution of the universe, measurement of velocities of moving objects etc.) are related to the propagation of signals in space or in space-time [1], [2]. The basis of experimental data received as results of observations of the Doppler effect or of the Hubble effect gives rise to theoretical considerations about the theoretical status of effects related to detecting signals from emitters moving relatively to observers carrying detectors in their laboratories.

2. In the classical (non-quantum) field theories different models of space-time have been used for describing the physical phenomena and their evolution. The 3-dimensional Euclidean space E_3 is the physical space used as the space basis of classical mechanics [3]. The 4-dimensional (flat) Minkowskian space $\overline{M}_4$ is used as the model of space-time in special relativity [4]. The (pseudo) Riemannian spaces V_4 without torsion are considered as models of space-time in general relativity [5], [6]. In theoretical gravitational physics (pseudo) Riemannian spaces without torsion as well as (pseudo) Riemannian spaces U_4 with torsion are proposed as space-time grounds for new gravitational theories. To the most sophisticated models of space-time belong the spaces with one affine connection and metrics [(L_n, g)-spaces] and the spaces with affine connections and metrics [$(\overline{L}_n, g)$-spaces].

The spaces with one affine connection and metrics [(L_n, g)-spaces] have affine connections whose components differ only by sign for contravariant and covariant tensor fields over a differentiable manifold M with dim $M = n$.

The spaces with affine connections and metrics have affine connections whose components differ not only by sign for contravariant and covariant tensor fields over a differentiable manifold M with dim $M = n$.

3. Recently, it has been shown [7], [8] that every differentiable manifold M ($dimM = n$) with affine connections and metrics [$(\overline{L}_n, g)$-spaces] [9] could be used as a model of space-time for the following reasons:

- The equivalence principle (related to the vanishing of the components of an affine connection at a point or on a curve) holds in $(\overline{L}_n, g)$-spaces [10]÷[12].

- $(\overline{L}_n, g)$-spaces have structures similar to these in (pseudo) Riemannian spaces without torsion [V_n-spaces] allowing for description of dynamic systems and the gravitational interaction [8].
- Fermi-Walker transports and conformal transports exist in $(\overline{L}_n, g)$-spaces as generalizations of these types of transports in V_n-spaces [13], [14].
- A Lorentz basis and a light cone could not be deformed in $(\overline{L}_n, g)$-spaces as it is the case in V_n-spaces.
- All kinematic characteristics related to the notions of relative velocity and of relative acceleration could be worked out in $(\overline{L}_n, g)$-spaces without changing their physical interpretations in V_n-spaces [8], [15]$\div$[18].
- $(\overline{L}_n, g)$-spaces include all types of spaces with affine connections and metrics used until now as models of space-time.

4. If a $(\overline{L}_n, g)$-space could be used as a model of space or of space-time the question arises how signals could propagate in a space-time described by a $(\overline{L}_n, g)$-space. The answer of this question has been given in [18], [19], and [20]. By that the signals are described by means of null (isotropic) vector fields. A signal could be defined as a periodical process transferred by an emitter and received by an observer (detector) [19]. A wave front could be considered as a signal characterized by its wave vector (null vector) as it is the case in the geometrical optics in a V_n-space [21]. All results are obtained on *purely kinematic basis* (s. [18], [22], [23]) without taking into account the dynamic reasons for the considered effect.

On the basis of the general results in the previous papers we can draw a rough scheme of the relations between the kinematic characteristics of the relative velocity and relative acceleration on the one side, and the Doppler effect and the Hubble effect on the other. Here the following abbreviations are used:

CM - classical mechanics
SRT - special relativity theory
GRT - general relativity theory
CRT - classical relativity theory [7], [8].

5. The considerations of the Doppler effect and of the Hubble effect show that the Doppler effect is derived in the physical theories (with exception of general relativity) as a result of the relative motion of an observer and an emitter, sending signals to the observer, from point of view of the proper frame of reference of the observer and its relations to the proper frame of reference of the emitter. On the other side, the Hubble effect could be considered as a result of the Doppler effect and the Hubble law assumed to be valid in the corresponding physical theory. In a rough scheme the relations between Doppler effect, Hubble effect, and Hubble law could be represented as follows:

	Relative motion characterized by		Doppler effect characterized as		Hubble effect characterized as		Hubble law characterized
CM	constant relative velocity	$\Rightarrow$	corollary	$\rightarrow$	corollary	$\Leftarrow$	by definition
SRT	constant relative velocity	$\Rightarrow$	corollary	$\rightarrow$	corollary	$\Leftarrow$	by definition
GRT	change of the metrics of space-time	$\Rightarrow$	corollary	$\rightarrow$	corollary	$\Leftarrow$	by definition of the metrics
CRT	relative velocity and relative acceleration	$\Rightarrow$	corollary	$\rightarrow$	corollary	$\leftarrow$	as corollary
		$\Downarrow$					$\Uparrow$
		$\Rightarrow$	$\longrightarrow$	$\rightarrow$	$\longrightarrow$	$\rightarrow$	$\Uparrow$

6. Since the Doppler effect and the Hubble effect as kinematic effects could be described by different theoretical schemes and models of space-time the rich mathematical tools of the spaces with affine connections and metrics, considered as models of space-time, are used for description of both the effects. The aim has been to work out a theoretical model of the Doppler effect and of the Hubble effect as corollaries only of the relative motion between emitter and observer determined by the kinematic characteristics of the relative velocity and the relative acceleration between emitter and observer from point of view of the proper frame of reference of the observer. For this task the $(n-1)+1$ formalism has been used related to the world line of an observer and its corresponding $n-1$ dimensional sub space interpreted as the observed space in the proper frame of the observer [22].

7. Our task in the present paper is to investigate the Doppler effect and the Hubble effect in the case of an auto-parallel motion of the observer and its influence on the frequency shift parameter. In section 2 the Doppler effect and shift frequency parameter are considered in the case of an auto-parallel motion of the observer. In Section 3 the Hubble effect and shift frequency parameter are considered for the same case. It is shown that by the use of the shift frequency parameter, considered locally in the proper frame of reference of an observer [22], we can directly determine the centrifugal (centripetal) relative velocity and acceleration as well as the Coriolis relative velocity and acceleration. Section 4 comprises some concluding remarks. Most of the details and derivation omitted in this paper could be found in [19] and in [20].

1.1 Abbreviation and symbols

- The vector field u is the velocity vector field of an observer: $u \in T(M)$, $\dim\ M = n$, $n = 4$.
- The contravariant vector field $v_z = \mp l_{v_z} \cdot n_\perp = H \cdot l_{\xi_\perp} \cdot n_\perp = H \cdot \xi_\perp$ is orthogonal to u and collinear to $\xi_\perp$. It is called centrifugal (centripetal) relative velocity.
- The function $H = H(\tau)$ is called Hubble function.

- The contravariant vector field $a_z = \mp l_{a_z} \cdot n_\perp = \overline{q} \cdot l_{\xi_\perp} \cdot n_\perp = \overline{q} \cdot \xi_\perp$ is orthogonal to u and collinear to $\xi_\perp$. It is called centrifugal (centripetal) relative acceleration.

- The function $\overline{q} = \overline{q}(\tau)$ is called acceleration function (parameter).

- The contravariant vector field $v_{\eta c} = \mp l_{v_{\eta c}} \cdot m_\perp = \overline{H}_c \cdot l_{\xi_\perp} \cdot m_\perp = \overline{H}_c \cdot \eta_\perp$ is orthogonal to u and to $\xi_\perp$. It is called Coriolis relative velocity.

- The function $\overline{H}_c = \overline{H}_c(\tau)$ is called Coriolis Hubble function.

- The contravariant vector field $a_{\eta_c} = \mp l_{a_{\eta c}} \cdot m_\perp = \overline{q}_{\eta c} \cdot l_{\xi_\perp} \cdot m_\perp = \overline{q}_{\eta c} \cdot \eta_\perp$ is orthogonal to u and to $\xi_\perp$. It is called Coriolis relative acceleration.

- The fuction $\overline{q}_{\eta c} = \overline{q}_{\eta c}(\tau)$ is called Coriolis acceleration function (parameter).

- $\overline{\omega}$ is the frequency of a signal emitted by an emitter at a time $\tau - d\tau$ of the proper time of the observer.

- ω is the frequency of a signal detected by the observer at a time τ of the porper time of the observer.

- $d\tau$ is the time interval in the proper frame of reference of the observer for which the signal propagates from the emitter to the observer (detector) at a space distance dl in the proper frame of reference of the observer.

2 Doppler effect and shift frequency parameter in the case of an auto-parallel motion of the observer

It has been shown [19], [20] that in a $(\overline{L}_n, g)$-space longitudinal (standard) and transversal Doppler effects could appear when signals are propagating from an emitter to an observer (detector) moving relatively to each other.

2.0.1 Longitudinal (standard) Doppler effect and the shift frequency parameter

1. Let us now consider the shift frequency parameter when the observer's world line is an auto-parallel trajectory, i.e. when the velocity vector u of the observer fulfills the equation

$$\nabla_u u = f \cdot .u \qquad , \qquad f \in C^\infty(M) \ .$$

Then, because of $\overline{g}[h_u(u)] = 0$,

$$a_\perp = \overline{g}[h_u(a)] = f \cdot \overline{g}[h_u(u)] = 0 \quad ,$$

$$(\nabla_u a)_\perp = \overline{g}[h_u(\nabla_u(f \cdot u))] = \overline{g}[h_u((uf) \cdot u + f \cdot a)] =$$
$$= (uf) \cdot \overline{g}[h_u(u)] + f \cdot \overline{g}[h_u(a)] = 0 \qquad .$$

Let a signal with frequency $\overline{\omega}$ is emitted by an emitter [19] at the time $\tau - d\tau$ and is received by an observer (detector) at the time τ.

Since $\overline{\omega} = \omega(\tau - d\tau)$ and $\omega = \omega(\tau)$, we can expand $\overline{\omega}$ in a Teylor row up to the second order of $d\tau$

$$\overline{\omega} = \omega(\tau - d\tau) \approx \omega(\tau) - \frac{d\omega}{d\tau}_{|\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\omega}{d\tau^2}_{|\tau} \cdot d\tau^2 + O(d\tau) \qquad .$$

Then

$$d\omega = \overline{\omega} - \omega \approx -\frac{d\omega}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\omega}{d\tau^2} \cdot d\tau^2 \qquad ,$$
$$\frac{d\omega}{\omega} = \frac{\overline{\omega} - \omega}{\omega} \approx -\frac{1}{\omega} \cdot \frac{d\omega}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{1}{\omega} \cdot \frac{d^2\omega}{d\tau^2} \cdot d\tau^2 \qquad .$$

On the other side, the general results in the case of an auto-parallel motion of the observer read [19], [20]

$$\frac{\overline{\omega} - \omega}{\omega} = \frac{d\omega}{\omega} = \omega \cdot (1 - \frac{\overline{l}_{v_z}}{l_{\xi_\perp}} + \frac{1}{2} \cdot \frac{dl}{l_u^2} \cdot \overline{l}_{a_z}) =$$
$$= \omega \cdot (1 - \frac{dl}{l_u} \cdot \frac{l_{v_z}}{l_{\xi_\perp}} + \frac{1}{2} \cdot \frac{dl^2}{l_u^2} \cdot \frac{l_{a_z}}{l_{\xi_\perp}}) \qquad .$$

Since

$$\frac{dl}{l_u} = d\tau \qquad , \qquad \frac{dl^2}{l_u^2} = d\tau^2 \qquad ,$$

we obtain the relations

$$\overline{\omega} = \omega \cdot (1 - \frac{l_{v_z}}{l_{\xi_\perp}} \cdot d\tau + \frac{1}{2} \cdot \frac{l_{a_z}}{l_{\xi_\perp}} \cdot d\tau^2) \qquad .$$

2. If we consider only infinitesimal changes of the frequency ω for the time interval $d\tau$, i.e. if $d\omega = \overline{\omega} - \omega$, we can express the shift parameter $z = (\overline{\omega} - \omega)/\omega$ as an infinitesimal quantity

$$z = \frac{d\omega}{\omega} = d(\log\ \omega) =$$
$$= d\overline{z} = -\frac{1}{\omega} \cdot \frac{d\omega}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{1}{\omega} \cdot \frac{d^2\omega}{d\tau^2} \cdot d\tau^2 =$$
$$= -\frac{l_{v_z}}{l_{\xi_\perp}} \cdot d\tau + \frac{1}{2} \cdot \frac{l_{a_z}}{l_{\xi_\perp}} \cdot d\tau^2 \qquad .$$

On the other side, $d\overline{z}$ could be considered as a differential of the function $\overline{z}$ depending on the proper time τ of the observer, i.e. $\overline{z} = \overline{z}(\tau)$. It is assumed

that $\overline{z}(\tau)$ has the necessary differentiability properties. The function $\overline{z}$ at the point $\overline{z}(\tau - d\tau)$ could be represented in Teylor row as

$$\overline{z}(\tau - d\tau) = \overline{z}(\tau) - \frac{d\overline{z}}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\overline{z}}{d\tau^2} \cdot d\tau^2 + O(d\tau) \quad .$$

Then $\overline{z}(\tau - d\tau)$ and $d\overline{z} = \overline{z}(\tau - d\tau) - \overline{z}(\tau)$ could be written up to the second order of $d\tau$ respectively as

$$\begin{aligned}
\overline{z}(\tau - d\tau) &= \overline{z}(\tau) - \frac{d\overline{z}}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\overline{z}}{d\tau^2} \cdot d\tau^2 \quad , \\
d\overline{z} &= \overline{z}(\tau - d\tau) - \overline{z}(\tau) = \\
&= -\frac{d\overline{z}}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\overline{z}}{d\tau^2} \cdot d\tau^2 = \\
&= -\frac{1}{\omega} \cdot \frac{d\omega}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{1}{\omega} \cdot \frac{d^2\omega}{d\tau^2} \cdot d\tau^2 = \\
&= -\frac{l_{v_z}}{l_{\xi_\perp}} \cdot d\tau + \frac{1}{2} \cdot \frac{l_{a_z}}{l_{\xi_\perp}} \cdot d\tau^2 \quad .
\end{aligned}$$

The comparison of the coefficients before $d\tau$ and $d\tau^2$ in the last (above) two expressions leads to the relations

$$\frac{d\overline{z}}{d\tau} = \frac{1}{\omega} \cdot \frac{d\omega}{d\tau} \quad , \qquad \frac{d^2\overline{z}}{d\tau^2} = \frac{1}{\omega} \cdot \frac{d^2\omega}{d\tau^2} \quad ,$$

$$\frac{d\overline{z}}{d\tau} = \frac{l_{v_z}}{l_{\xi_\perp}} \quad , \qquad \frac{d^2\overline{z}}{d\tau^2} = \frac{l_{a_z}}{l_{\xi_\perp}} \quad .$$

The vector $\xi_\perp$ could be chosen as a unit vector, i.e. $l_{\xi_\perp} = 1$, equal to the vector $n_\perp$ showing the direction to the emitter from point of view of the observer. Then

$$\frac{d\overline{z}}{d\tau} = l_{v_z} \quad , \qquad \frac{d^2\overline{z}}{d\tau^2} = l_{a_z} \quad .$$

Therefore, if we can measure the change (variation) of the shift frequency parameter $d\overline{z}$ in a time interval $d\tau$ we can find the centrifugal (centripetal) relative velocity and centrifugal (centripetal) relative acceleration of the emitter with respect to the observer. The above relations appear as direct way for a check-up of the considered theoretical scheme of the propagation of signals in spaces with affine connections and metrics. On the other side, the explicit form of l_{v_z} and l_{a_z} as functions of the kinematic characteristics of the relative velocity and relative acceleration could lead to conclusions of the properties of the space-time model used for description of the physical phenomena. The same relations could lead to more precise assessment of the Hubble function H and the acceleration function $\overline{q}$ at a given time.

2.0.2 Transversal Doppler effect and the shift frequency parameter

In analogous way we can find the relations between the absolute values of the Coriolis relative velocity and the Coriolis relative acceleration and the shift frequency parameter in the case of auto-parallel world line of the observer.

1. The relation between the frequency $\overline{\omega}$ of the emitted signals and the frequency ω of the detected signals reads [19], [20]

$$\overline{\omega} = \omega \cdot (1 - \frac{dl}{l_u} \cdot \frac{l_{v_{\eta c}}}{l_{\xi_\perp}} + \frac{1}{2} \cdot \frac{dl^2}{l_u^2} \cdot \frac{l_{a_{\eta c}}}{l_{\xi_\perp}}) =$$
$$= \omega \cdot (1 - \frac{l_{v_{\eta c}}}{l_{\xi_\perp}} \cdot d\tau + \frac{1}{2} \cdot \frac{l_{a_{\eta c}}}{l_{\xi_\perp}} \cdot d\tau^2) \quad .$$

The shift frequency parameter has the form

$$z_c = \frac{\overline{\omega} - \omega}{\omega} = -\frac{l_{v_{\eta c}}}{l_{\xi_\perp}} \cdot d\tau + \frac{1}{2} \cdot \frac{l_{a_{\eta c}}}{l_{\xi_\perp}} \cdot d\tau^2 = d\overline{z}_c =$$
$$= -\frac{d\overline{z}_c}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\overline{z}_c}{d\tau^2} \cdot d\tau^2 + O(d\tau) \approx$$
$$\approx -\frac{d\overline{z}_c}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\overline{z}_c}{d\tau^2} \cdot d\tau^2 \ .$$

The comparison of the coefficients before $d\tau$ and $d\tau^2$ in the two expressions

$$z_c = \frac{\overline{\omega} - \omega}{\omega} = -\frac{l_{v_{\eta c}}}{l_{\xi_\perp}} \cdot d\tau + \frac{1}{2} \cdot \frac{l_{a_{\eta c}}}{l_{\xi_\perp}} \cdot d\tau^2 = d\overline{z}_c =$$
$$\approx -\frac{d\overline{z}_c}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\overline{z}_c}{d\tau^2} \cdot d\tau^2$$

leads to the relations

$$\frac{d\overline{z}_c}{d\tau} = \frac{l_{v_{\eta c}}}{l_{\xi_\perp}} \quad , \qquad \frac{d^2\overline{z}_c}{d\tau^2} = \frac{l_{a_{\eta c}}}{l_{\xi_\perp}} \quad .$$

The vector $\xi_\perp$ could be chosen as a unit vector, i.e. $l_{\xi_\perp} = 1$, equal to the vector $n_\perp$ showing the direction to the emitter from point of view of the observer. Then

$$\frac{d\overline{z}_c}{d\tau} = l_{v_{\eta c}} \quad , \qquad \frac{d^2\overline{z}_c}{d\tau^2} = l_{a_{\eta c}} \quad .$$

Therefore, if we can measure the change (variation) of the shift frequency parameter $d\overline{z}_c$ in a time interval $d\tau$ we can find the Coriolis relative velocity and Coriolis relative acceleration of the emitter with respect to the observer. The above relations appear as direct way for a check-up of the considered theoretical scheme of the propagation of signals in spaces with affine connections and metrics. On the other side, the explicit form of $l_{v_{\eta c}}$ and $l_{a_{\eta c}}$ as functions of the kinematic characteristics of the relative velocity and relative acceleration could lead to conclusions of the properties of the space-time model used for description of the physical phenomena. The same relations could lead to more precise assessment of the Hubble function H_c and the acceleration function $\overline{q}_{\eta c}$ at a given time.

2.1 Hubble effect and shift frequency parameter in the case of an auto-parallel motion of the observer

It has been shown [19], [20] that in a $(\overline{L}_n, g)$-space longitudinal (standard) and transversal Hubble effects could appear when signals are propagating from an emitter to an observer (detector) moving relatively to each other.

2.1.1 Longitudinal (standard) Hubble effect and the shift frequency parameter

By the use of the relations [19], [20]

$$l_{v_z} = \mp H \cdot l_{\xi_\perp} \qquad , \qquad l_{a_z} = \mp \overline{q} \cdot l_{\xi_\perp} \qquad ,$$

$$\frac{d\overline{z}}{d\tau} = \frac{l_{v_z}}{l_{\xi_\perp}} \qquad , \qquad \frac{d^2\overline{z}}{d\tau^2} = \frac{l_{a_z}}{l_{\xi_\perp}} \qquad ,$$

we can find the Hubble function H and the acceleration function (parameter) $\overline{q}$ respectively as

$$\frac{d\overline{z}}{d\tau} = \mp H \qquad , \qquad \frac{d^2\overline{z}}{d\tau^2} = \mp \overline{q} \qquad .$$

2.1.2 Transversal Hubble effect and the shift frequency parameter

By the use of the relations [19], [20]

$$l_{v_{\eta c}} = \mp \overline{H}_c \cdot l_{\xi_\perp} \qquad , \qquad l_{a_{\eta c}} = \mp \overline{q}_{\eta c} \cdot l_{\xi_\perp} \qquad ,$$

$$\frac{d\overline{z}_c}{d\tau} = \frac{l_{v_{\eta c}}}{l_{\xi_\perp}} \qquad , \qquad \frac{d^2\overline{z}_c}{d\tau^2} = \frac{l_{a_{\eta c}}}{l_{\xi_\perp}} \qquad ,$$

we can find the transversal Hubble function H_c and the transversal acceleration function (parameter) $\overline{q}_{\eta c}$ respectively as

$$\frac{d\overline{z}_c}{d\tau} = \mp \overline{H}_c \qquad , \qquad \frac{d^2\overline{z}_c}{d\tau^2} = \mp \overline{q}_{\eta c} \qquad .$$

3 Conclusion

In the present paper we have considered the Doppler effect and the Hubble effect for the case of auto-parallel motion of the observer in a $(\overline{L}_n, g)$-space and their relations to the shift frequency parameters corresponding to the longitudinal and transversal effects. It is shown that these effects lead to direct check-up of the theoretical scheme and could be used for finding out the relative centrifugal (centripetal) velocities and accelerations as well as the relative Coriolis velocities and accelerations of moving astronomical objects from point of view of the proper frame of reference of an observer (detector).

The Doppler effects and the Hubble effects are considered on the grounds of purely kinematic considerations. It should be stressed that the Hubble functions

H and $\overline{H}_c$ are introduced on a purely kinematic basis related to the notions of relative centrifugal (centripetal) velocity and to the notions of Coriolis velocities respectively. They could be found directly by the use of the measurements of the shift frequency parameters. It should be noted that $\overline{H}_c$ does not exists in the Einstein theory of gravitation. The dynamic interpretations of H and $\overline{H}_c$ in a theory of gravitation depend on the structures of the theory and the relations between the field equations and on both the functions. In this paper it is shown that notions the specialists use to apply in theories of gravitation and cosmological models could have a good kinematic grounds independent of any concrete classical field theory. Doppler effects, and Hubble effects could be used in mechanics of continuous media and in other classical field theories in the same way as the standard Doppler effect is used in classical and special relativistic mechanics.

References

[1] Weinberg S 1972 *Gravitation and cosmology: Principles and applications of the general theory of relativity* (New York: John Wiley and Sons)

[2] Unzicker A 2003 Galaxies as Rotating Buckets - a Hypothesis on the Gravitational Constant Based on Mach's Principle. (*Preprint* ArXiv gr-qc/03 08 087)

[3] Javorskii B M and Detlaff A A 1977 *Handbook of Physics* (Moscow: Nauka,) (in Russian) 532 552 533

[4] Tonnelat M A 1962 *Foundations of electromagnetism and the theory of relativity* (Moscow: Inostr. lit.) (in Russian)

[5] Anderson J A 1967 *Principles of relativity physics* (New York: Academic Press) 270-271

[6] Misner Ch W, Thorne K S and Wheeler J A 1973 *Gravitation.* (San Francisco: W H Freeman and Company). Russian translation 1977: Vol 1., Vol. 2., Vol. 3. (Moscow: Mir)

[7] Manoff S 2002 *Geometry and Mechanics in Different Models of Space-Time*: *Geometry and Kinematics.* (New York: Nova Science Publishers) Parts 2 - 3

[8] Manoff S 2002 *Geometry and Mechanics in Different Models of Space-Time*: *Dynamics and Applications.* (New York: Nova Science Publishers) Parts 1 - 2

[9] Manoff S 1999 Spaces with contravariant and covariant affine connections and metrics *Physics of elementary particles and atomic nucleus (PEPAN)* [Russian Edition: **30** 5 1211-1269], [English Edition: **30** 5 527-549]

[10] Iliev B Z 1996 Normal frames and the validity of the equivalence principle: I. Cases in a neighborhood and at a point *J. Phys. A: Math. Gen.* **29** 6895-6901

[11] Iliev B Z 1997 Normal frames and the validity of the equivalence principle: II. The case along paths *J. Phys. A: Math. Gen.* **30** 4327-4336

[12] Iliev B Z 1998 Normal frames and the validity of the equivalence principle: III. The case along smooth maps with separable points of self-interaction *J. Phys. A: Math. Gen.* **31** 1287-1296

[13] Manoff S 2000 Fermi derivative and Fermi-Walker transports over $(\overline{L}_n, g)$-spaces *Intern. J. Mod. Phys.* **A 13** 5 679-695

[14] Manoff S 1998 Conformal derivative and conformal transports over $(\overline{L}_n, g)$-spaces *Intern. J. Mod. Phys.* **A 15** 25 4289-4308

[15] Manoff S 2002 Mechanics of continuous media in $(\overline{L}_n, g)$-spaces. 1. Introduction and mathematical tools. (*Preprint ArXiv*: gr-qc / 02 03 016

[16] Manoff S 2002 Mechanics of continuous media in $(\overline{L}_n, g)$-spaces. 2. Relative velocity and deformations. (*Preprint ArXiv*: gr-qc / 02 04 003

[17] Manoff S 2002 Mechanics of continuous media in $(\overline{L}_n, g)$-spaces. 3. Relative accelerations. (*Preprint ArXiv*: gr-qc / 02 03 016

[18] Manoff S 2003 Centrifugal (centripetal), Coriolis velocities, accelerations, and Hubble law in spaces with affine connections and metrics (*Preprint ArXiv*: gr-qc / 02 12 038); *Central European J. of Physics* **4** 669-694

[19] Manoff S 2003 Propagation of signals in spaces with affine connections and metrics (*Preprint ArXiv*: gr-qc / 03 09 050)

[20] Manoff S 2004 *e*Propagation of signals in spaces with affine connections and metrics as models of space-time [to appear in *Physics of elementary particles and atomic nuclei (PEPAN)*, **4** (2004)]

[21] Stephani H 1977 *Allgemeine Relativitaetstheorie* (Berlin: VEB Deutscher Verlag d. Wissenschaften) 75-76

[22] Manoff S 2001 Frames of reference in spaces with affine connections and metrics (*Preprint ArXiv* gr-qc/99 08 061) *Class. Quantum Grav.* **18** 6 1111-1125

[23] Manoff S 1995 Kinematics of vector fields In *Complex Structures and Vector Fields.* eds. Dimiev St., Sekigawa K. (Singapore: World Scientific) 61-113